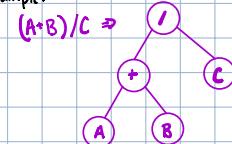


CONNECTION

You may have heard of parse trees: rooted trees that represent the syntactic structure of a string according to some context-free grammar. Even if you haven't heard of parse trees, they can make conversions between prefix, infix, and postfix easier and are somewhat intuitive. For example:



As you might expect, a preorder traversal results in prefix notation, an inorder traversal results in infix notation, and a postorder traversal results in postfix notation.

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EXAMPLE

Convert the following postfix expression into prefix. If $A=7$, $B=9$, $C=3$, $D=4$, $E=6$, and $F=8$, evaluate the following postfix expression

$$AAB+ CAB/-/* \quad \text{bracket off operators/operands}$$

$$A*((A+B)/(C-(A/B))) \quad \text{use parentheses to make infix}$$

$$*A/ + AB - C / AB \quad \text{use infix groupings to translate to postfix}$$

$$A*((A+B)/(C-(A/B))) \quad \text{translate postfix back to infix to check}$$

Translate the formula for the volume of a sphere from infix to prefix

$$\frac{4}{3}\pi R^3 \quad \text{or} \quad 4\pi R^3/3$$

$$* * 4 3 \pi ^ R 3 \quad /* * 4 \pi ^ R 3 3$$

b

FIXING IT UP

prefix, infix, postfix

a notation guide

1

EXAMPLE

If $A=7$, $B=9$, $C=3$, $D=4$, $E=6$, and $F=8$, evaluate the following postfix expression

$$ABCDEA - + * / + F E - +$$

$$(A+(B/(C*(D+(E-A)))))+(F-E))$$

$$(7+(9/(3*(4+(6-7)))+(8-6))$$

$$(7+(9/(3*(4+(6-1)))+2)$$

$$(7+(9/(3*3))+2)$$

$$(7+(9/9))+2$$

$$(7+1)+2$$

$$8+2$$

$$10$$

s

I see these problems like an onion: I look at the different bracketed "layers" and attack the innermost bracket first.



POSTFIX

Postfix notation, sometimes called reverse Polish notation, is quite similar to prefix notation. The difference is that instead of placing operators immediately before their operands, postfix places operators immediately after their operands. Postfix is used for the same reason prefix is: ease of conversion into a tree. Below is an example:

$$AB+C/ \rightarrow \text{first add } A \text{ and } B, \text{ then}$$

divide that sum by C.

You might notice that in prefix/postfix notation, any time you have two operands next to each other you have an "inner" term that can be grouped

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PRACTICE

Convert the following infix expression into prefix: $A^3 B + (D-E)$

C

$$+ * ^ A 3 B C - D E$$

Convert the following postfix expression into prefix: $AA B+ CA B/ - / *$

$$* A / + AB - / ABC$$

Find all integer values of X for which the following postfix expression has a value of 0:
 $* + X 4 - 6 X$

$$X = -4, 6$$

8

PREFIX

Prefix notation, sometimes also called Polish notation, is a way of writing expressions in which operators directly precede their operands, meaning that sometimes we need additional information to determine the order in which operations were intended to be performed. The order of operations (PEMDAS) is used to establish precedence of operations and we can place parentheses to clarify what we mean when we are using infix notation. Below is an example:

$$(A+B)/C \rightarrow \text{first add } A \text{ and } B, \text{ then}$$

divide that sum by C.

Note that we had to use parentheses to eliminate ambiguity in the order the operations were intended to be performed

2

divide that sum by C

3

